

Clock Metrics

WSTS-2025 Tutorial Session

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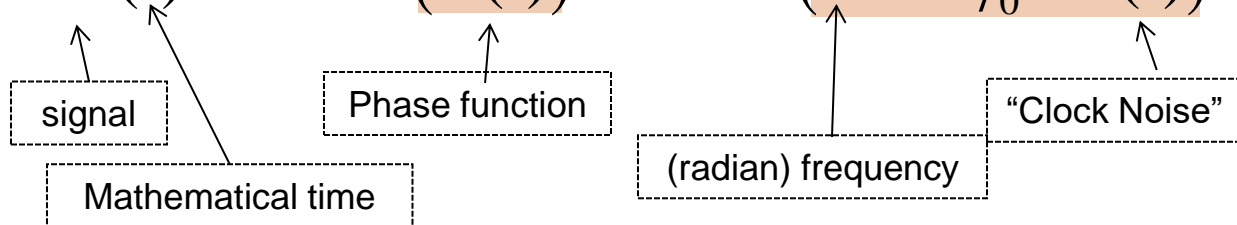
Synchronization Metrics (Performance)

- ▶ Mathematical Model
- ▶ Fundamental Clock Concepts and Metrics
 - Time Error (TE) and Time Interval Error (TIE)
 - MTIE
 - TDEV
- ▶ Relationship between TDEV, Spectrum, and MTIE
 - Use of TDEV to identify noise type
 - Using TDEV for guidance on loop bandwidths



Common Mathematical Models

$$clock(t) = A \cdot \cos(\Phi(t)) = A \cdot \cos(\omega \cdot t + \phi_0 + \varepsilon(t))$$



- A : Amplitude of signal. Does not figure in timing metrics.
- ϕ_0 : Initial phase. Depends on choice of time origin. Usually assumed to be 0.
- $\varepsilon(t)$: Can be further decomposed into different categories such as frequency error, frequency drift, and random noise components
- ideal periodic signal: $\Phi(t)$ is a linear function of t ($\varepsilon(t) \equiv 0$)

Continuous time view

$$x(t) = a_0 + y \cdot t + \left(\frac{1}{2}\right) \cdot D \cdot t^2 + \phi(t)$$

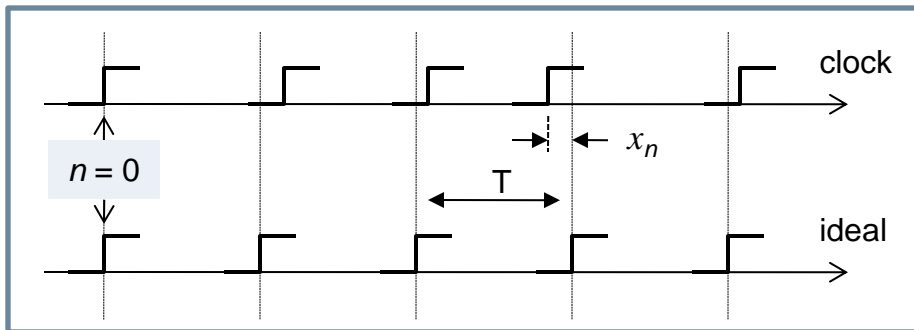
Discrete time view

$$x(nT_s) = a_0 + y \cdot nT_s + \left(\frac{1}{2}\right) \cdot D \cdot (nT_s)^2 + \phi(nT_s)$$

Time Error
Models



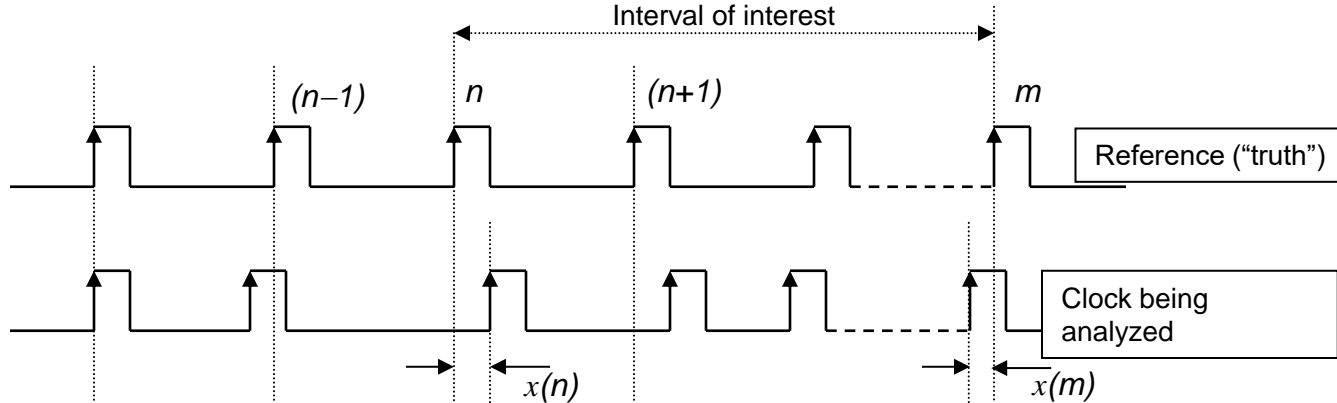
Clock Metrics – Basics: Time Error



- ▶ Clock signals are (almost) periodic (nominal period $\sim T$)
- ▶ Time Error (Phase Error):
 - Edges do not line up – *phase error* (expressed in time units)
- ▶ Time Error Sequence : $\{x_n\}$ or $\{x(n)\}$
 - *All clock metrics derived from time error sequence*
 - Note: the time error varies “slowly” so we can divide down to a convenient rate (However: careful when dividing down – aliasing)
 - **Common assumption: $x_0 = 0$.**



Time Interval Error



- Consider an interval of interest (e.g. 100m dash)
- Duration measured by ideal clock ("truth") : $(m - n) \cdot T_S$
- *Error* in measurement of same interval by clock being analyzed:

$$TIE(m, n) = x(m) - x(n)$$



Clock Metrics – MTIE and TDEV

MTIE

A measure of peak-to-peak excursion expected within a given interval, τ (τ is a parameter). The observation interval is scanned with a moving window of duration τ and $MTIE(\tau)$ is the maximum excursion.

Given a set of N observations $\{x(k); k=0,1,2,\dots,(N-1)\}$, with underlying sampling interval τ_0 , let $\tau = n \cdot \tau_0$ (“window” = n samples; $n = 1,2,\dots,N$).

Peak-to-peak excursion over n samples starting with sample index i is the worst-case TIE in this interval of n samples:

$$peak-to-peak(i) = \left\{ \max_{k=i}^{k=i+n-1} x(k) - \min_{k=i}^{k=i+n-1} x(k) \right\}$$

$MTIE(n)$, or $MTIE(\tau)$, is the largest value of this peak-to-peak excursion:

$$MTIE(n) = \max_{i=0}^{N-n} \left\{ \max_{k=i}^{k=i+n-1} x(k) - \min_{k=i}^{k=i+n-1} x(k) \right\}$$



Clock Metrics – MTIE and TDEV

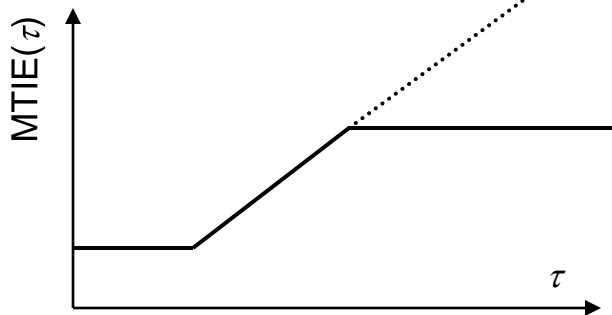
MTIE

MTIE is a useful indicator of the size of buffers and for predicting buffer overflows and underflows.



Buffer size $> \text{MTIE}(\tau)$ implies that overflow/underflow unlikely in any interval $< \tau$

Buffer size $= \text{MTIE}(\tau)$ implies that overflow/underflow occurs approx. every τ seconds



Observations regarding MTIE:

- monotonically increasing with τ
- linear increase indicates freq. offset
- for small τ , $\text{MTIE}(\tau) \leftrightarrow \text{jitter}$
- for medium τ , $\text{MTIE}(\tau) \leftrightarrow \text{wander}$
- for large τ , indicates whether “locked” (zero-slope)



Clock Metrics – MTIE and TDEV

TDEV

A measure of stability expected over a given observation interval, τ (τ is a parameter).

Given a set of N observations $\{x(k); k=0,1,2,\dots,(N-1)\}$ with underlying sampling interval τ_0 , let $\tau = n \cdot \tau_0$ (“window” = n samples; $n = 1,2,\dots,N$).

Note: $x(k) \Leftrightarrow x_k$

$$\sigma_x(\tau) = TDEV(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)} \sum_{j=0}^{N-3n} \left[\sum_{i=j}^{n+j-1} (x_{i+2n} - 2x_{i+n} + x_i) \right]^2}$$

for $n=1,2,\dots,\left\lfloor \frac{N}{3} \right\rfloor$

Conventional
Definition

Second-order difference

N-point averaging

Sum of squares

- TVAR = square of TDEV
- Modified Allan Deviation (MDEV)

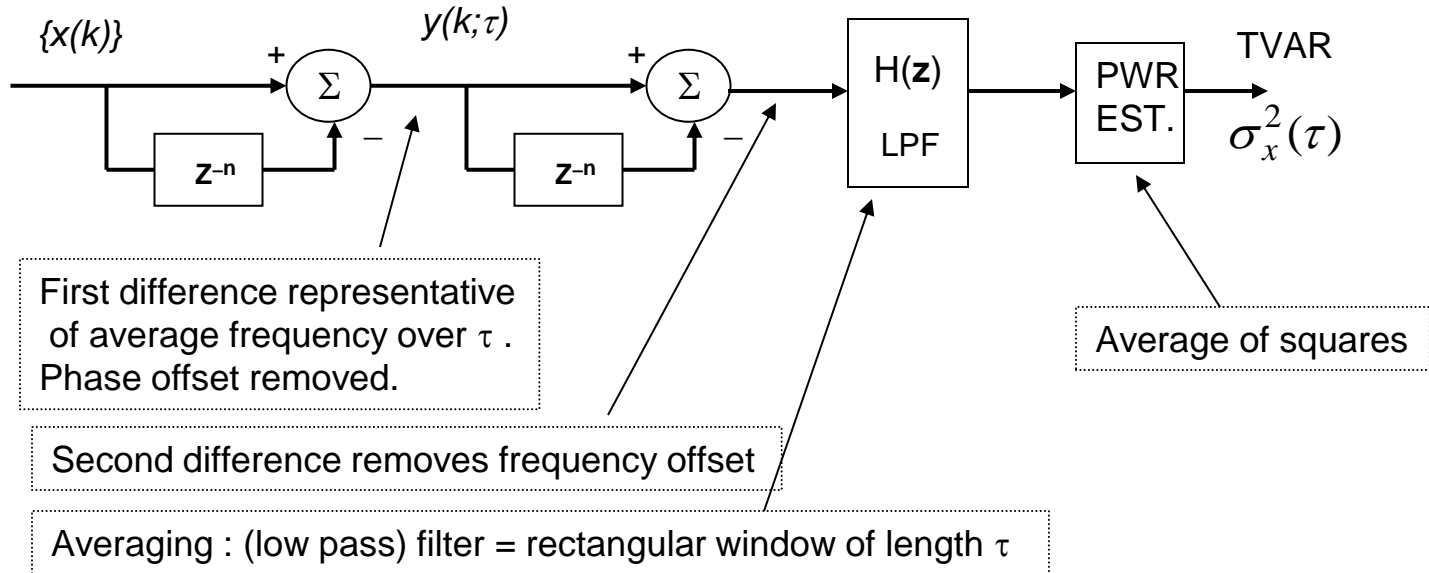
$$\sigma_y(\tau) = \frac{\sqrt{3}}{\tau} \sigma_x(\tau)$$

- TDEV suppresses initial phase and frequency offset and quantifies the strength of the frequency drift and noise components {i.e. $\varepsilon(t)$ }
- TDEV provides guidance on the noise process type



Clock Metrics – MTIE and TDEV

Signal Processing Interpretation of TDEV and TVAR



Noise Types, TDEV, Spectrum & MTIE

TDEV (and MDEV) for different noise types

Noise Process	Dependence of TDEV(τ) on τ	Dependence of MDEV(τ) on τ
White PM	$\tau^{-(1/2)}$	$\tau^{-(3/2)}$
Flicker PM	τ^0	τ^{-1}
Random Walk PM = White FM	$\tau^{+(1/2)}$	$\tau^{-(1/2)}$
Flicker FM	τ^{+1}	τ^0
Random Walk FM	$\tau^{+(3/2)}$	$\tau^{+(1/2)}$

Spectrum ($S(f)$) for different noise types

Noise Process	Spectrum Type (power)
White PM	f^0
Flicker PM	f^{-1}
Random Walk PM = White FM	f^{-2}
Flicker FM	f^{-3}
Random Walk FM	f^{-4}

When linear frequency drift dominates, TDEV(τ) behaves as τ^2

Approximate relationship between TDEV and power spectrum:
(For guidance purposes only)

$$S_x(f) \approx \left(\frac{0.75}{f} \right) \cdot \left(\sigma_x \left(\frac{0.3}{f} \right) \right)^2$$

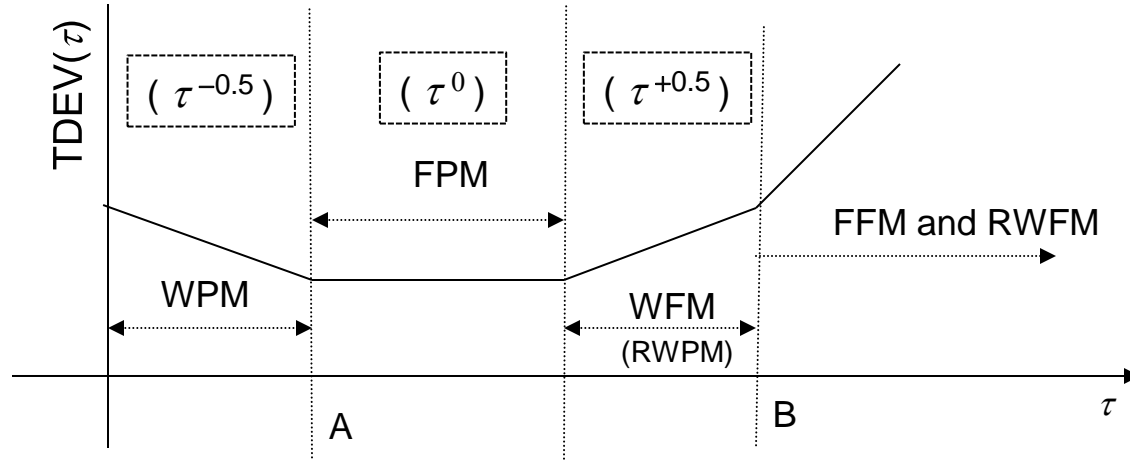
$$\sigma_x(\tau) \approx \sqrt{\left(\frac{1}{2.5 \cdot \tau} \right) \cdot S_x \left(\frac{0.3}{\tau} \right)}$$

Approximate relationship between TDEV and MTIE:
 $K_1 \sim 0.75$; $K_2 \sim 0.3$ (excludes effect of transients)

$$M_x(\tau) \leq 7 \cdot \sqrt{4 \cdot K_1 \cdot \int_0^{f_0} \frac{1}{f} \cdot TVAR \left(\frac{K_2 \cdot f_0}{f} \right) \cdot \sin^2(n\pi f \tau_0) \cdot df}$$



Implication of $TDEV(\tau)$ versus τ



“Phase coherence” for up to A sec.

⇒ Keep PLL time constants less than A sec.

Phase Flicker Floor

“Frequency coherence” for up to B sec.

⇒ Keep FLL time constants less than B sec.

Frequency Flicker Floor



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Thank You

Questions, comments, suggestions?

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